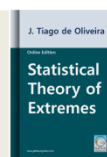




Statistical Theory of Extremes

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Part 4

Multivariate Extremes

Exercises

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- 4.1. Consider the EMS sequence $\{Z_k\}$. Obtain the expression of $\text{Prob}\{\min(Z_1, \dots, Z_k) \geq z\} = Q_k(z, z|\theta)$

where

$$Q_k(x, y|\theta) = \text{Prob}\left\{\min_{1 \leq i \leq k-1} (Z_i, \dots, Z_{k-1}) \geq x, Z_k \geq y\right\}$$

$$= Q_{k-1}(x, \max(x, y - \log \theta)) \wedge (y - \log(1 - \theta)).$$

Obtain its expression when $x < y - \log \theta$ and $x > y - \log \theta$.

- 4.2. Consider the general EMS sequence $X_k = \lambda + \delta Z_k$ and the seek estimators of $(\lambda, \delta, \theta)$. We have $\min_{1 \leq i \leq k} (X_i - X_{i-1}) \xrightarrow{P} \delta \log \theta$ which gives an (over-) estimator of $\Delta = \delta \log \theta$. Using this result obtain, supposing $\Delta^* = \Delta$, an estimator of (λ, δ) .
- 4.3. Translate maxima results to minima results and vice-versa; use, in particular, the relation between the distribution function and the survival function.

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- 4.4. Study the processes of oldest ages of death for men and women in Sweden (Table.1 and Tables 1 + 2) see [Exercise of Part 3](#), trying to fit any of the random sequences that may have trend and/or oscillations. In particular they may be considered as sliding processes of maxima, with Gumbel margins.
- 4.5. Consider a reduced extremal process $Z(t), t > 0$ observed at (non-random) instants $(0 <) t_1 < t_2 < \dots < t_n < \dots$. The best (least squares) predictor of $Z(t_{n+1})$ ($t_n < t_{n+1}$) of the form $Z_1^*(t) = Z(t_n) + a$ has $a = \log(t_{n+1}/t_n)$. The best (i.e., least squares) linear predictor of $Z(t_{n+1})$ based only on $Z(t_n)$ is of the form $Z^{**}(t_{n+1}) = \alpha + \beta Z(t_n)$; the minimization of $MSE = M(Z^{**}(t_{n+1}) - Z(t_{n+1}))^2$ leads to

$$Z^{**}(t_{n+1}) = \gamma + \log t_{n+1} + \rho(t_n/t_{n+1}) (Z(t_n) - \gamma + \log t_n).$$

They are both unbiased (i.e., with the same mean value) and

$$MSE(Z^*(t_{n+1})) = \frac{\pi^2}{3} (1 - \rho(t_n/t_{n+1})) \text{ and } MSE(Z^{**}(t_{n+1})) = \frac{\pi^2}{6} (1 - \rho^2(t_n/t_{n+1})).$$

Note that $Z^{**}(t_{n+1})$ can be compared with the simpler linear predictor $Z^*(t_{n+1}) = Z(t_n) + \log \frac{t_{n+1}}{t_n}$. The efficiency is

$$\frac{MSE(Z^*(t_{n+1}))}{MSE(Z^{**}(t_{n+1}))} = \frac{1 + \rho(t_n/t_{n+1})}{2} < 1.$$

None of the predictors is invariant for linear transformations, and so they can't be used to predict the general extremal processes $X(t) = \lambda + \delta Z(t)$.

- 4.6. Consider the extremal processes $X(t) = \lambda + \delta Z(t), t \geq 0$, where $Z(t)$ is the reduced extremal process. Suppose observations are made at (non-random) instants $(0 <) t_1 < t_2 < \dots < t_n < \dots$.

Obtain the expression of the quasi-linear predictor of $X(t_{n+1})$ when $X(t_i), i = 1, \dots, n$, are known. The quasi-linear predictor of $X(t_{n+1})$ based on the last two observations $X(t_{n-1})$ and $X(t_n)$ ($\geq X(t_{n-1})$) is, obviously,

$$X^*(t_{n+1}) = X(t_n) + \beta(X(t_n) - X(t_{n-1})).$$

The best (least squares) predictor of $X(t_{n+1})$, minimizing

$$\text{MSE}(X^*(t_{n+1})) = M(X^*(t_{n+1}) - X(t_{n+1}))^2 ,$$

is given by

$$\beta = \frac{M((X(t_{n+1}) - X(t_n))(X(t_n) - X(t_{n-1})))}{M((X(t_n) - X(t_{n-1}))^2)}$$

and

$$\text{MSE}(X^*(t_{n+1})) = M((X(t_{n+1}) - X(t_n))^2) - \beta^2 M((X(t_n) - X(t_{n-1}))^2) ,$$

which can be expressed in mean values and covariance of the process.

- 4.7. Define an extremal process of Weibull minima, using the conversion between the Weibull distribution of minima and the Gumbel distribution of maxima.
- 4.8. Consider a max-compound Poisson stochastic process, $X(t) = \max\{Y_i\}$, where $N(t)$ is a Poisson process with intensity v and $Y_0, Y_1, \dots, Y_k, \dots$, is a sequence of independent random variables such that Y_0 has the distribution function $G_0(x)$ and $Y_j (j \geq 1)$ have the distribution function $G(x)$. Show that $\text{Prob}\{X(t) \leq x\} = G_0(x) \exp\{-vt(1 - G(x))\}$ and that for no choice of (G_0, G) can we have $\text{Prob}\{X(t) \leq x\} = \Lambda(\alpha(t) + \beta(t)x)$.
- 4.9. Analyse the same question for max-filtered Poisson processes and max-renewal point processes.
