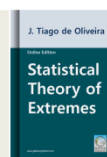




Statistical Theory of Extremes

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Part 3

Multivariate Extremes

Annex 5

On the Quadrants Method

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Consider a sample of n i.i.d. random pairs $\{(x_1, y_1), \dots, (x_n, y_n)\}$ whose distribution function is $F(x, y|\theta)$, where θ is a (unique) dependence parameter and the margins $F(x, +\infty|\theta)$ and $F(+\infty, y|\theta)$ are parameter-free.

Let us denote by ξ and η the medians the margins ($F(\xi, +\infty|\theta) = F(+\infty, \eta|\theta) = 1/2$), by N_1, N_2, N_3, N_4 ($N_1 + N_2 + N_3 + N_4 = n$) the sample cardinals $N_1 = \#(x_i > \xi, y_i > \eta)$, $N_2 = \#(x_i \leq \xi, y_i > \eta)$, $N_3 = \#(x_i \leq \xi, y_i \leq \eta)$ and $N_4 = \#(x_i > \xi, y_i \leq \eta)$, and by $p_1(\theta), p_2(\theta), p_3(\theta), p_4(\theta)$ the probabilities $p_1(\theta) = \text{Prob}(X > \xi, Y > \eta) = p(\theta)$, $p_2(\theta) = \text{Prob}(X \leq \xi, Y > \eta) = 1/2 - p(\theta)$, $p_3(\theta) = F(\xi, \eta|\theta) = p(\theta)$ and $p_4(\theta) = \text{Prob}(X > \xi, Y \leq \eta) = 1/2 - p(\theta)$.

The maximum likelihood estimator of θ based on (N_1, N_2, N_3, N_4) is given by the equation $(\theta^{**}) = \frac{(N_1 + N_3)}{2n}$; denoting by $N = N_1 + N_3$ we have $p(\theta^{**}) = N/2n$.

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As $\sqrt{n} \frac{N/n-2p(\theta)}{\sqrt{2p(\theta)(1-2p(\theta))}}$ is asymptotically standard normal we know, by the δ -method — see [Tiago de Oliveira \(1982\)](#) — that $\sqrt{n} \frac{p'(\theta)}{\sqrt{p(\theta)(1-2p(\theta))}} (\theta^{**} - \theta)$ is also asymptotically standard normal, as well as $\frac{2n^{3/2} p'(\theta^{**})(\theta^{**} - \theta)}{\sqrt{n(n-N)}}$.

If we consider $F(x, y|\theta) = \Lambda(x, y|\theta)$ we have $\xi = \eta = -\log \log 2$ and we get
 $p(\theta) = (\xi, \eta|\theta) = \exp\{(-e^{-\xi} + e^{-\eta})k(\eta - \xi|\theta)\} = \exp(-2 \log 2 \cdot k(0|\theta))$.
 Thus the estimator θ^{**} is given by the equation $k(0|\theta^{**}) - \frac{\log(2n/N)}{\log 4}$. But, for any (one-parameter) model, we have
 $\bar{k} = \sup_{\theta} k(0|\theta)$, $\underline{k} = \inf_{\theta} k(0|\theta)$; suppose that $\bar{k} = k(0|\bar{\theta})$ and $\underline{k} = k(0|\underline{\theta})$ have unique solutions: then we shall truncate the estimator and take $\theta^{**} = \bar{\theta}$ if $N/2n > \bar{k}$ and $\theta^{**} = \underline{\theta}$ if $N/2n < \underline{k}$.

A test of independence vs. dependence (i.e. if $p(0) = 1/4$ vs. $p(0) > 1/4$) is given by the rejection region $\sqrt{n}(2N/n) - 1 \geq \lambda_{\alpha}$ where $\lambda_{\alpha} = N^{-1}(1 - \alpha)$, $N(\cdot)$ being the standard normal distribution function. This acts as a test independent of the model *but* only based on N_1, N_2, N_3, N_4 : essentially it can confirm dependence.

We can, evidently, extend the method to use other quantiles in the margins or with estimated parameters in the margins.

This method is due to [Gumbel and Mustafi \(1967\)](#), but presented here with modifications.

References

- [Gumbel, E. J. and Mustafi, C. K., 1967. Some analytical properties of bivariate extremal distributions. *J. Amer. Statist. Assoc.*, 62, 569-58.](#)
[Tiago de Oliveira, J., 1982. A definition of estimator efficiency in k-parameter case. *Inst.Statist. Math.*, 34 A, 411-421.](#)
